MULTI-STAGE DECODING

OF

MULTI-LEVEL MODULATION CODES[1]

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Abstract

This paper investigates various types of multi-stage decoding for multi-level modulation codes. It is shown that if the componenet codes of a multi-level modulation code and types of decoding at various stages are chosen properly, high spectral efficiency and large coding gain can be achieved with reduced decoding complexity. Particularly, it is shown that the difference in performance between the suboptimum multi-stage soft-decision maximum likelihood decoding of a modulation code and the single-stage optimum soft-decision decoding of the code is very small, only a fraction of dB loss in SNR at BER of 10^{-6}.

1. Introduction

Coded modulation is a technique of combining coding and bandwidth efficient modulation to produce modulation (or signal space) codes for achieving reliable data transmission without compromising bandwidth efficiency [1-4]. Over the last eight years, a great deal of research effort has been expended in constructing good bandwidth efficient modulation codes. Among all the proposed methods for constructing modulation codes, the most powerful one is the multi-level construction method [2,3,5-9]. This method allows us to construct modu-

lation codes systematically with arbitrarily large minimum **squared Euclidean distance** from Hamming distance component codes (**binary** or **nonbinary, block** or **convolutional**) in conjunction with proper **bits-to-signal** mapping through **signal set partitioning**. If the component codes are chosen properly, the resultant multi-level modulation code not only has good minimum squared Euclidean distance but is also rich in structural properties such as: **regularity, linear structure, phase symmetry** and **trellis structure**. These structural properties simplify the error performance analysis, encoding and decoding implementations, and resolution of carrier-phase ambiguity. A major advantage of multi-level modulation codes is that these codes can be decoded in **multiple stages** with component codes decoded sequentially stage by stage, with decoded information passed from one stage to another stage. Since component codes are decoded one at a time, it is possible to take advantage of the structure of each component code to simplify the decoding complexity and reduce the number of computations at each stage. As a result, the overall complexity and number of computations needed for decoding a multi-level modulation code will be greatly reduced. This allows us to achieve high reliability, large coding gain and high spectral efficiency with reduced decoding complexity.

2. Multi-Stage Decoding of Multi-Level Modulation Codes

There are four possible types of multi-stage decoding:

(1) **Multi-stage Soft-decision Maximum Likelihood Decoding** - Each stage of decoding is a soft-decision maximum likelihood decoding;

(2) **Multi-stage Hard-decision Maximum Likelihood Decoding** - Each stage of decoding is a hard-decision maximum likelihood decoding;

(3) **Multi-stage Bounded-distance Decoding** - Each decoding stage is a bounded-distance decoding based on a certain distance measure, e.g., Hamming distance; and

(4) **Hybrid Multi-stage Decoding** - Mixed types of decoding are used among the stages.

With the multi-stage soft-decision maximum likelihood decoding, each component code of a multi-level modulation code is chosen to have trellis structure and is decoded with the soft-decision Viterbi decoding algorithm. Since the decoding at each stage depends on the decoded information from the previous decoding stages, there is a likelihood of error propagation. As a result, the overall decoding is not optimum even though the decoding at each stage is optimum. It is a suboptimum decoding. However, the error propagation effect can be made negligibly small, if the first few component codes(mostly the first component code) of a multi-level modulation code are powerful. Based on our analysis and simulation of the error performance of several efficient multi-level modulation codes, we find that the difference in performance between the suboptimum multi-stage decoding and the single-stage

optimum decoding is very small, only a fraction of dB loss in SNR at the BER(block or bit error rate) of 10^{-6}.

With the multi-stage hard-decision maximum likelihood decoding, each component code is also chosen to have trellis structure, but is decoded with the hard decision Viterbi decoding algorithm. This type of multi-stage decoding further simplifies the decoding complexity, however there is a 2-2.5 dB **loss** in SNR compared to the optimum soft-decision decoding. Even with some loss in SNR, the multi-stage hard-decision maximum likelihood decoding still achieves significant coding gain over an uncoded system with the same spectral efficiency based on our computations and simulations of error performance of some multi-level modulation codes.

With the multi-stage bounded distance decoding, component codes of a multi-level modulation code are decoded with bounded-distance decoding based on either Euclidean or Hamming distance measure. If a component code is binary, its minimum squared Euclidean distance is linearly propotional to its minimum Hamming distance. As a result, it can be decoded based on its minimum Hamming distance. In this case, algebraic or majority-logic decoding may be used. Results show that if the first-level component code is a low-rate powerful code and the other component codes are high-rate code, the multi-stage bounded distance decoding can also achieve significant coding gain over an uncoded system without any bandwidth expansion and with greatly reduced decoding complexity.

The hybrid multi-stage decoding provides an excellent trade-off between coding gain and decoding complexity. With this scheme, the lower-level decoding stages (specially the first-level decoding) are soft-decision maximum likelihood decoding using Viterbi decoding algorithm and the higher-level decoding stages are hard-decision maximum likelihood or bounded distance decoding. Based on our computation and simulation of error performance of some multi-level modulation codes, we find that the hybrid multi-stage decoding has less than one dB loss in coding gain compared to the optimum decoding.

A very natural architecture for a multi-stage decoder is the **pipeline architecture**. For a multi-level modulation code with m component codes, the decoder is organized to decode m received vectors in pipeline process. While the decoder is decoding the m-th component vector of the earliest received vector in the pipe, it is also decoding the $(m-1)$-th component vector of the next received vector in the pipe, ..., and the first component vector of the most recent received vector. This pipeline architecture speeds up the decoding process.

3. Examples

Consider a basic 3-level 8-PSK block modulation code of length 32 with the following three component codes: (1) C_1 is the $(32,6)$ Reed-Muller code with Hamming distance $\delta_1 = 16$; (2) C_2 is the $(32,26)$ Reed-Muller code with Hamming distance $\delta_2 = 4$; and (3) C_3 is the $(32,31)$ even parity check code with Hamming distance $\delta_3 = 2$. This basic 3-level 8-

PSK modulation code, $C = C_1 * C_2 * C_3$, has minimum squared Euclidean distance $D[C] = 8$ and spectral efficiency $\eta[C] = 63/32 = 1.966$. This code achieves 6 dB asymptotic coding gain over the uncoded QPSK with optimal decoding. The first component code C_1 has a 4-section 16-state trellis, the second component code C_2 also has a 4-section 16-state trellis, and the third component code C_3 has a 32-section 2-state trellis. The overall modulation code $C = C_1 * C_2 * C_3$ has a 512-state trellis. To perform the single-stage optimum decoding for the overall code, we need to build a soft-decision Viterbi decoder with 512 states which is quite complex and expensive. However, with the multi-stage soft-decision maximum likelihood decoding for this code, we need only two 16-state and one 2-state Viterbi decoders (a total of 34 states) for the three component codes. The total complexity is much less than that of a single 512-state Viterbi decoder for optimum decoding. The error performance of the code is shown in **Figure 1**. We see that, with multi-stage soft-decision maximum likelihood decoding, there is almost 5 dB in **real coding gain** over the uncoded QPSK at block-error-rate (BER) 10^{-6}, which is only 1 dB away from the 6 dB asymptotic coding gain. If optimum decoding is performed, the real coding gain of the code over the uncoded QPSK is 5.25 dB at BER $= 10^{-6}$. We see that there is an **excellent trade-off** between the error performance and decoder complexity.

Figure 1 also includes the error performance of the above 3-level 8-PSK modulation code using 3-stage hard-decision maximum likelihood decoding. We see there is a 2.3 dB loss in SNR at the BER of 10^{-6} compared with the 3-stage soft-decision suboptimum decoding. However, there is still 2.7 dB coding gain over the uncoded QPSK system with very little bandwidth expansion. With the 3-stage hard-decision decoding, the decoding complexity is further reduced.

As a second example, consider a 3-level 8-PSK block modulation code of length 64 with the following component codes: (1) C_1 is the second order (64,22) Reed-Muller code with minimum Hamming distance $\delta_1 = 16$; (2) C_2 is the 4-th order (64,57) Reed-Muller code with minimum Hamming distance $\delta_2 = 4$; and (3) C_3 is the (64,63) even parity check code with minimum Hamming distance $\delta_3 = 2$. This 3-level 8-PSK modulation code, $C = C_1 * C_2 * C_3$, has minimum squared Euclidean distance $D[C] = 8$ and spectral efficiency $\eta[C] = 142/64 = 2.22$. The first component code has a 4-section trellis diagram with 2^{10} states, the second component code has a 4-section trellis diagram with 2^5 states, and the third component code has a 2-state trellis diagram. The overall code has a 4-section trellis diagram with 2^{16} states. Decoding this code with the single-stage soft-decision maximum likelihood decoding using Viterbi algorithm is prohibitively complex. However, with 3-stage soft-decision maximum likelihood decoding, this code achieves a 4.5 dB coding gain over the uncoded QPSK system at the block-error-rate 10^{-6} (see Figure 2) with a big reduction in decoding complexity(from a complexity of 65536 states to a complexity of 1058 states). In fact, this coding gain is achieved with a bandwidth reduction. With the 3-stage hard-decision bounded distance decoding, the code also achieves significant coding gain over the uncoded QPSK system with bandwidth reduction(see Figure 2). There is a 2.2 dB loss in coding gain compared with

the 3-stage soft-decision maximum likelihood decoding, however the decoding complexity is greatly reduced. Note that the first component code is majority-logic decodable and the second component code is simply a distance-4 extended Hamming code which can be easily decoded. To improve the performance while still keeping the complexity down, we may use the hybrid multi-stage decoding in which the first component code is decoded with the hard-decision bounded distance decoding, and the second and third componenet codes are decoded with the soft-decision maximum likelihood decoding using the Viterbi algorithm.

4. Conclusion

In our examples, we used block modulation codes to demonstrate the effectiveness of the multi-stage decoding. The multi-stage decoding can be applied to decode the multi-level trellis modulation codes. This type of decoding for multi-level modulation code really offers the best of three worlds, spectral efficiency, coding gain(or error performance), and decoding complexity.

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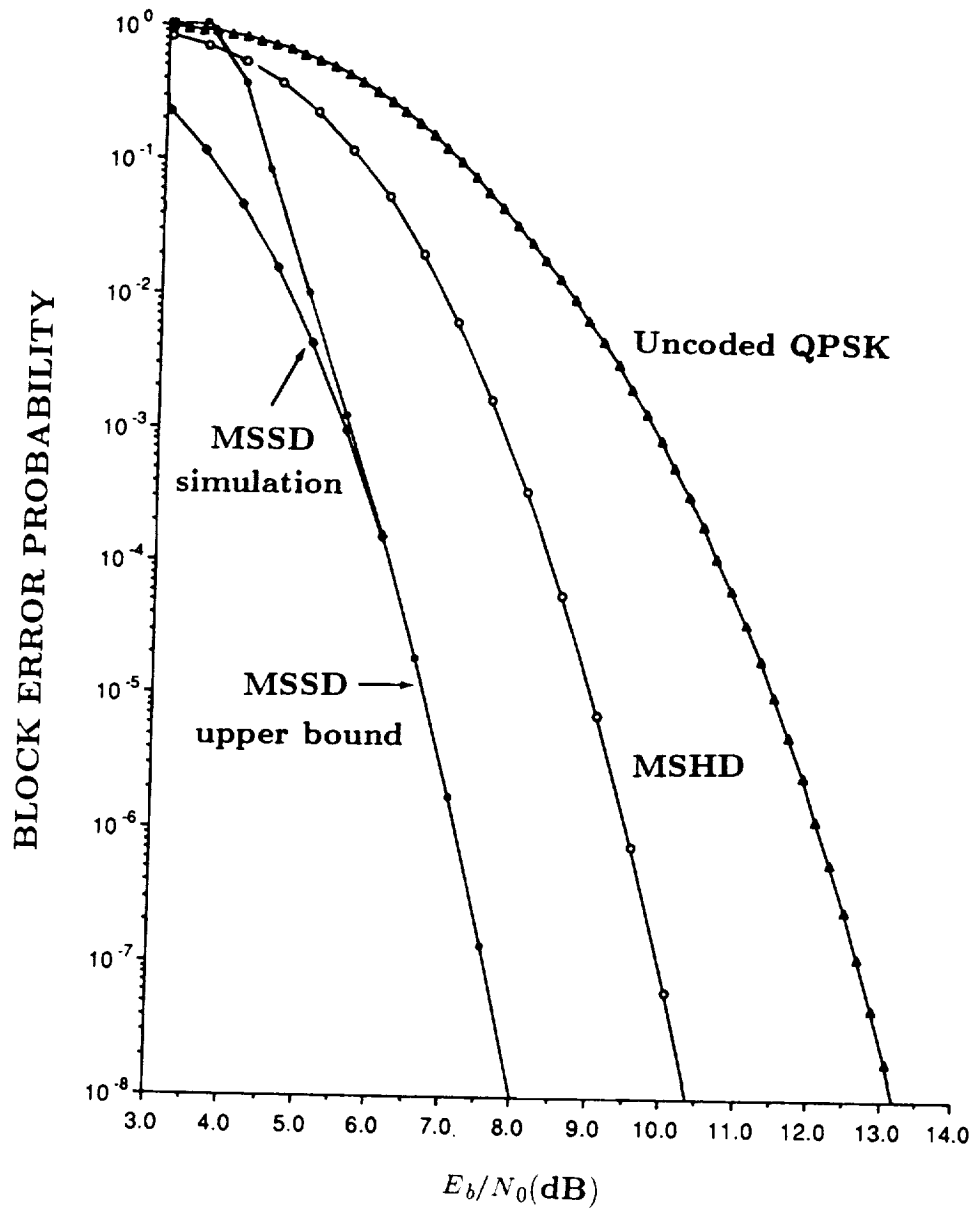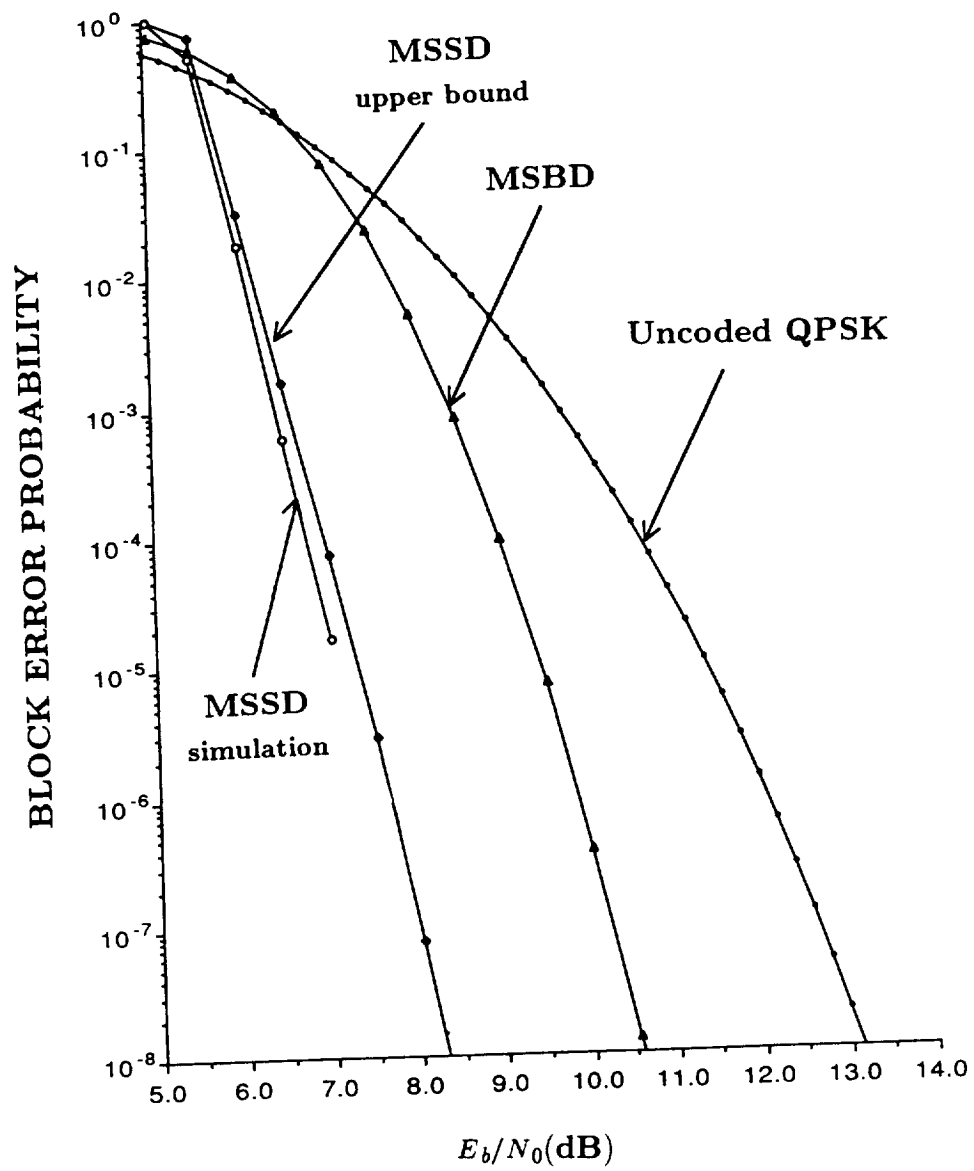
Figure 1

Figure 2